# Page From a Physicist's Notebook

## Stopping on a Dime
## (or perhaps on a loonie!)

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The basic slide-to-stop equation shows that a vehicle's stopping distance increases dramatically (actually quadratically) with initial speed. This has some very interesting – and some very dangerous – consequences for motor vehicle collisions.

For example, suppose that a vehicle, originally travelling at 50 km/h, can just stop before hitting an obstruction (another vehicle, a tree – or a pedestrian!) Now consider that, if that same vehicle were to be travelling at 55 km/h – just 5 km/h faster – the impact speed in the inevitable collision would be around 30 km/h!

So, it isn't always possible to "stop on a dime". Sometimes, considerable time and distance is required to avoid involvement in serious crashes. So, perhaps rather than stopping on a dime, we need to consider being able to stop on a loonie!!

**Slide-to-stop**

In a previous "*Page from a Physicist's Notebook*"[1] we reviewed the derivation of the slide-to-stop equation using a number of basic concepts in physics.

**From basic principles…**

Definition of speed

$$v = \frac{d}{t}$$

Acceleration due to gravity

$$g = 9.81 \text{ m/s}^2$$

In particular, we derived two results, namely one for the vehicle speed measured in m/s:

$$v = 4.43\sqrt{\mu d} \quad \text{m/s} \qquad (1)$$

and the associated version for the vehicle speed being measured in km/h:

$$v = 15.9\sqrt{\mu d} \quad \text{km/h} \qquad (2)$$

where:

$\mu$ = coefficient of friction
d = vehicle stopping distance (m)

| v (km/h) | $v^2$ | $\mu$ | 252.8 $\mu$ | d (m) | | |
|---|---|---|---|---|---|---|
| 20 | 400 | 0.7 | 176.96 | 2.26 | | |
| 30 | 900 | 0.7 | 176.96 | 5.09 | | |
| 40 | 1600 | 0.7 | 176.96 | 9.04 | 4 x 2.26 = | 9.04 |
| 50 | 2500 | 0.7 | 176.96 | 14.13 | | |
| 60 | 3600 | 0.7 | 176.96 | 20.34 | | |
| 70 | 4900 | 0.7 | 176.96 | 27.69 | | |
| 80 | 6400 | 0.7 | 176.96 | 36.17 | 16 x 2.25= | 36.17 |
| 90 | 8100 | 0.7 | 176.96 | 45.77 | | |
| 100 | 10000 | 0.7 | 176.96 | 56.51 | | |
| 110 | 12100 | 0.7 | 176.96 | 68.38 | | |
| 120 | 14400 | 0.7 | 176.96 | 81.37 | | |

**Table 1. Vehicle stopping distance as a function of speed**

Another way to look at this equation is to consider the stopping distance as a function of vehicle speed. Re-arranging Equation 2 (by squaring both sides, and then dividing both sides by 252.8μ) gives:

$$v^2 = 15.9^2 \mu d$$

$$d = v^2 / 252.8\mu$$

The importance of this equation is that vehicle stopping distance can be seen to vary as the square of the vehicle's speed.

Thus, there is not a linear relationship between stopping distance and speed. Rather this is a quadratic function such that the stopping distance becomes vastly longer with increased speed.

Simple slide-to-stop calculations, as shown in Table 1, illustrate the point. In all cases, a coefficient of friction of 0.7 (typical for hard braking on a dry asphalt roadway surface) is assumed.

Note that regular (10 km/h) increases in initial vehicle speed result in dramatically longer stopping distances.

For example, increasing the speed from just 20 km/h to 30 km/h more than doubles the required stopping distance (2.26 m to 5 m).

The quadratic nature of the relationship can be seen by examining the stopping distances as the vehicle speed is doubled.

At 20 km/h a stopping distance of 2.26 m is required. If the speed is doubled to 40 km/h, the stopping distance increases by a factor of four to about 9 m. If the speed is doubled again to 80 km/h, i.e. increased four times from 20 km/h, the stopping distance required is about 36 m, and is therefore increased sixteen times over that required for 20 km/h.

This quadratic relationship between initial vehicle speed and stopping distance is the reason why so many drivers experience problems in panic braking situations. They think that they should be able to stop in much shorter distances than are actually required.

However, there is another interesting – although extremely dangerous – aspect of the quadratic nature of braking distance with speed. This is the residual (impact) speed of a vehicle that has insufficient braking distance to come to a stop before reaching a hazard.

**Vehicle speed vs. distance under hard braking**

Prior to any event in which a driver brakes hard in an attempt to avoid a collision, the driver (a) needs to recognize that there is a hazard, and (b) must take their foot off the accelerator and press down firmly on the brake pedal. The combination of the time to perceive the hazard and the time to react to the situation by physically transferring the pedal effort from the accelerator to the brake, is the perception-reaction time ($t_{p-r}$).

During this period of perception-reaction, there is no hard braking (the driver hasn't yet reacted by applying the vehicle's brakes). Consequently, throughout the perception-reaction time period, the vehicle merrily continues to travel forward at its original (constant) speed.

Once the brake pedal is applied hard (in a "panic stop"), the vehicle starts to decelerate and will eventually come to a halt – provided that there is sufficient stopping distance. Should this not be the case, a collision will occur and the vehicle will have slowed down to its impact speed.

The calculation methodology used to determine the distance travelled in such circumstances is, therefore, a combination of a constant-speed phase (perception-reaction), and a constant-acceleration phase (deceleration due to hard-braking).

We will use the equations governing constant speed and constant acceleration to examine the distances travelled and the vehicle speeds involved in such situations.

**(a) Perception-reaction distance**

As noted above, before any braking occurs, the vehicle travels at constant speed during the driver's perception-reaction time.

Speed is defined as distance travelled divided by time taken:

$$v = \frac{d}{t}$$

where:

$v$ = vehicle speed (m/s)
$d$ = distance travelled (m)
$t$ = time taken (s)

Re-arranging this equation (multiply both sides by t) gives:

$$d = v\,t \qquad (3)$$

Suppose that we have a vehicle initially travelling at 50 km/h and a driver with a perception-reaction time of 1.5 seconds

Equation 3 gives the distance travelled by the vehicle in the 1.5 s period for perception-reaction:

$$d_{p-r} = v\,t_{p-r}$$

where

$v$ = 50 km/h = 50 / 3.6 = 13.9 m/s
$t_{p-r}$ = 1.5 s

$d_{p\text{-}r} = 13.9 \times 1.5 = 20.8$ m

So, we can see that, at an initial speed of 50 km/h, the vehicle will travel almost 21m before any braking (deceleration) occurs.

**(b) Distance travelled under hard braking**

Once the driver applies the brakes hard, in a "panic stop" situation, Equation 2 defines the event.

$$v = 15.9\sqrt{\mu d} \quad \text{km/h} \qquad (2)$$

However, in this instance, we are interested in the distance travelled as the vehicle brakes hard and comes to a stop. So, re-arranging the above equation gives

[Square both sides]

$$v^2 = 15.9^2 \ \mu\, d$$

$$v^2 = 252.8\ \mu\, d$$

[Divide both sides by 252.8μ]

$$d = \frac{v^2}{252.8\ \mu} \qquad (4)$$

where:

- $d$ = distance travelled (m)
- $v$ = initial speed (km/h)
- $\mu$ = coefficient of friction

In our example, the vehicle's initial speed is assumed to be 50 km/h. Let's further assume that the vehicle's tires and the road surface have a coefficient of friction of 0.7

Equation 4 gives the braking distance required to bring the vehicle to a complete stop from its initial speed of 50 km/h.

$$d_{braking} = \frac{v^2}{252.8\ \mu}$$

$$d_{braking} = \frac{50^2}{252.8 \times 0.7} = \frac{2500}{176.96}$$

$$d_{braking} = 14.1 \text{ m}$$

**(c) Total stopping distance**

The total required stopping distance is the distance travelled during the perception-reaction period ($d_{p\text{-}r}$) and the distance travelled during the hard-braking phase ($d_{braking}$).

$$d_{stopping} = d_{p\text{-}r} + d_{braking}$$

$$d_{stopping} = 20.8 + 14.1$$

$$d_{stopping} = 34.9 \text{ m}$$

Thus, given our assumptions for the (average) driver's perception-reaction time, and the available coefficient of friction (typical for a dry, asphalt-paved roadway), a vehicle initially travelling at 50 km/h will require almost 35 m in order for the driver to bring the vehicle to a complete stop.

**Vehicle travelling at 55 km/h**

Now, let's consider a vehicle, initially travelling at 55 km/h, that has only 34.9 m of available stopping distance.

Our original vehicle, travelling at 50 km/h, could just stop in this distance. Clearly, the faster vehicle cannot stop in the same distance. The question becomes at what speed will it still be travelling after it has covered the available 34.9 m stopping distance?

The same principles of constant speed and constant acceleration that we used above will apply to our calculation. However, we need to carefully consider the various phases of the new calculation. This is especially the case for the braking phase since the vehicle won't actually be in a slide-to-stop situation, rather it will slow down from its initial speed to its impact speed.

**(a) Perception-reaction phase**

In order to be able to compare apples with apples, let's keep the driver's perception-reaction time at 1.5 s.

The initial speed of the vehicle is now:

$$v = 55 \text{ km/h} = 55 / 3.6 = 15.3 \text{ m/s}$$

Consequently, Equation 3 gives us the distance travelled during the driver's perception-reaction period as:

$$d_{p\text{-}r} = v\, t_{p\text{-}r} = 15.3 \times 1.5$$

$$d_{p\text{-}r} = 22.9 \text{ m}$$

**(b) Braking phase**

The vehicle has now travelled along 22.9 m of the available 34.9 m of stopping distance. At this point, the vehicle is still travelling at its initial speed of 55 km/h, just as the driver brakes hard.

The available braking distance is now only:

$$d_{braking} = 34.9 – 22.9 = 12 \text{ m}$$

Note that for this vehicle, travelling at 55 km/h, and with only 12 m of braking distance available before it hits something, we do not have a slide-to-stop situation.

The vehicle is unable to stop in the available distance. There will be a crash and we wish to calculate the impact speed, i.e. the final speed at the end of the braking phase of the event.

Consequently, we need an equation that will relate the initial and final speeds along the braking path to the actual braking distance.

In the earlier "*Page from a Physicist's Notebook*"[1] we noted that the principle of conservation of energy applies such that the loss in the vehicle's kinetic energy as the vehicle slows down from its initial to its final speed is equal to the work done by the frictional force over the braking distance.

The equation governing this situation is:

$$\tfrac{1}{2}\, m v_1^2 – \tfrac{1}{2}\, m v_2^2 = \mu m g d \quad (5)$$

where:

$m$ = vehicle mass (kg)
$v_1$ = initial speed (m/s)
$v_2$ = final speed (m/s)
$\mu$ = coefficient of friction
$g$ = gravitational acceleration (9.81 m/s²)
$d$ = braking distance (m)

Equation 5 can be simplified to:

$$v_2^2 = v_1^2 – 2\mu g d$$

so that

$$v_2 = \sqrt{v_1^2 – 2\mu g d} \qquad (6)$$

In our current situation, the vehicle is travelling at an initial speed of 55 km/h and undergoes hard braking for a distance of 12 m so that:

$v_1 = 55$ km/h = 15.3 m/s
$\mu = 0.7$
$g = 9.81$ m/s$^2$
$d_{braking} = 12$ m

$$v_2 = \sqrt{15.3^2 - 2 \times 0.7 \times 9.81 \times 12}$$

$$v = \sqrt{233.4 - 164.8}$$

$$v = \sqrt{68.6} = 8.28 \text{ m/s}$$

$$v = 8.28 \times 3.6 = 29.8 \text{ km/h}$$

Thus, a vehicle initially travelling at 55 km/h will still be moving at about 30 km/h at a point where a vehicle initially travelling at 50 km/h would have been able to come to a complete halt.

**Speed-Distance Graph**

This result may surprise the lay person. A speed difference of just 5 km/h upstream can lead to a speed difference of 30 km/h at the end of the same conditions of hard braking! But, it shouldn't come as any surprise to those with a little knowledge of the underlying physics and mathematics. Faster vehicles travel further during the perception-reaction phase and require considerably greater braking distances than do vehicles initially travelling at slower speeds.

The effect can be clearly demonstrated by conducting a series of speed-distance calculations for a number of different initial speeds and charting the results on a graph.

While the individual calculations may be performed as indicated above, it is much more convenient (and efficient!) to program the calculations into a computer spreadsheet and have the spreadsheet program graph the results. An example of such calculations is shown in Figure 1.

The graphs in Figure 1 each show a vehicle travelling at a constant speed for a period of 1.5 s, followed by a reduction in the travel speed as the vehicle decelerates under hard braking (for which $\mu = 0.7$)

The negative speeds shown at the end of each curve should be ignored. These are a mathematical artifact resulting from regularly-spaced distances (1 m intervals) being used to compute the points on the graph while the vehicles actually come to rest (speed = 0) at distances that fall between the plotted points.

For example, our initial braking calculation showed that the total stopping distance for a vehicle with an initial speed of 50 km/h was 34.9 m. This corresponds to the green curve in Figure 1. The curve crosses the x-axis at 34.9 m, but points on the graph are only plotted for d=34 and d=35 m. Clearly the calculated "speed" of the vehicle at the latter point is meaningless since the vehicle has already stopped. The final point is merely included on the graph in order to show where the vehicle's speed actually reaches zero (i.e. crosses the x-axis).

Note that, once braking commences, the vehicle's speed falls off relatively gradually at first, then decreases at an ever-increasing rate, with the final large reductions in speed occurring close to the end of the stopping distance. This curve is a quadratic function for speed vs. distance as we have noted in the foregoing equations.

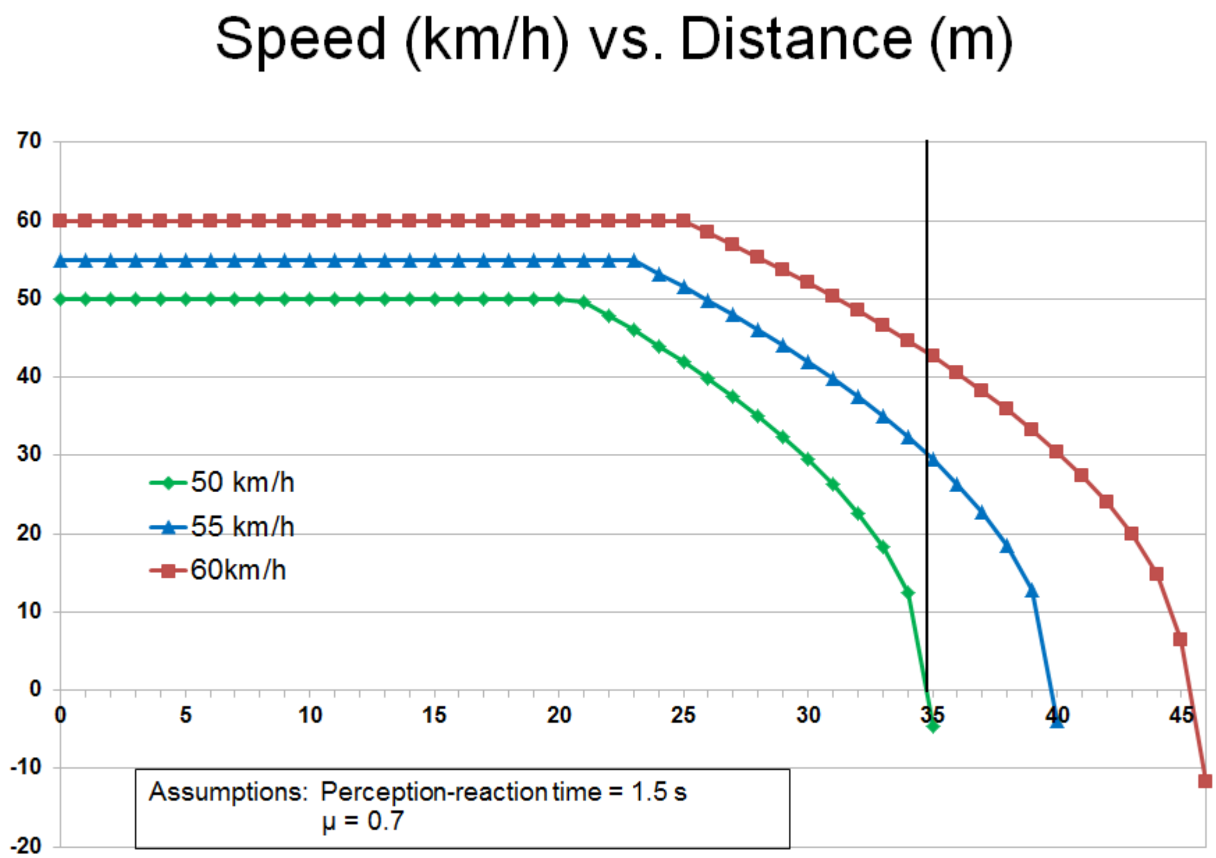


**Figure 1. Vehicle Speed vs. Distance Under Hard Braking**

The graph demonstrates the dramatic effect of the speed-squared nature of kinetic energy. Note, for example, on the green curve ($v_0$ = 50 km/h), that in the first 5 m of braking, the vehicle's speed drops by about 8 km/h. In the next 5 m of braking, the speed drops by about 12 km/h. But, in the final 5 m of braking, the vehicle's speed drops by approximately 30 km/h!

The vertical black line represents the required stopping distance of 34.9 m for a vehicle travelling at 50 km/h. Moving up this line to the blue curve shows that, as we calculated, a vehicle initially travelling at a speed of 55 km/h would still be moving at a speed of about 30 km/h after travelling this same distance.

The red curve shows that, a vehicle initially travelling at 60 km/h, just 10 km/h more than our 50 km/h example, would come into collision at a speed of about 43 km/h!

Clearly, an increase in initial speed of just a few km/h can have a large effect on impact speed in the event that a driver has insufficient stopping distance available in which to bring the vehicle to a halt in the face of a collision hazard.

Furthermore, such greater impact speeds can have significant consequences for the vehicle occupants in crashes with other vehicles or fixed objects, and are of even greater concern for collisions with pedestrians.

**Traffic Safety Promotion**

The concepts described above have been adopted by a number of Australian jurisdictions for use in public service announcements to promote traffic safety.

In one commercial [2], two vehicles are seen approaching a tractor-trailer that is travelling at right angles to their path. One vehicle almost comes to a stop before coming into a very minor collision with the side of the truck. The other vehicle, initially travelling just 5 km/h faster, crashes hard into the left-rear corner of the trailer.


In the second commercial [3], a pedestrian (the "pizza-guy") crosses the road and moves into the path of an on-coming vehicle – with disastrous results! A second vehicle, moving 10 km/h slower, is able to brake to a stop and merely give the pedestrian a scare.

These videos use the science that we have just reviewed to create a "hard-hitting" safety message, and graphically demonstrate the potential consequences of just a few extra km/h of speed.

**References**

1. German A; Page From a Physicist's Notebook: Work is mad - it takes energy!; *Proc. CMRSC-XV*; Fredericton, New Brunswick; June 5-8, 2005

2. Speed vs. Impact PSA
http://www.youtube.com/watch?v=bpJhf3qoOk4

3. TAC - 10 KPH Less (Australia)
http://www.youtube.com/watch?v=Ctkqd6hYMy8

**Author**

Dr. Alan German is a Chartered Physicist who was involved in the in-depth investigation and analysis of real-world crashes for over 25 years. He has lectured on the physics of crashes on a number of collision reconstruction courses.

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